My Presentation

And Some Things About It

John Doe

Institute of Physics
University of São Paulo

February 31, 2019

Summary

A Silly Idea

Ordinary Differential Equations

$$\frac{\mathrm{d}}{\mathrm{d}x} y(x) + \frac{1}{CR} y(x) = 0$$

$$\frac{\mathrm{d}^2}{\mathrm{d}x^2} y(x) + \gamma \frac{\mathrm{d}}{\mathrm{d}x} y(x) + \omega_0^2 y(x) = f(x)$$

Ordinary Differential Equations

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Livin' La Vida Loca

$$\frac{\mathrm{d}^2}{\mathrm{d}x^2}y(x) + \gamma\frac{\mathrm{d}}{\mathrm{d}x}y(x) + \omega_0^2 y(x) = f(x)$$

$$\Downarrow$$

$$\left[\frac{\mathrm{d}^2}{\mathrm{d}x^2} + \gamma\frac{\mathrm{d}}{\mathrm{d}x} + \omega_0^2\right]y(x) = f(x)$$

$$\Downarrow$$

$$y(x) = \frac{f(x)}{\frac{\mathrm{d}^2}{\mathrm{d}x^2} + \gamma\frac{\mathrm{d}}{\mathrm{d}x} + \omega_0^2}$$

Livin' La Vida Loca

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Livin' La Vida Loca

Pandora's Box

$$f(x) \longrightarrow \boxed{\mathcal{F}} \longrightarrow \hat{f}(\xi)$$

Pandora's Box

$$(f+\alpha g)(x) \longrightarrow \boxed{\mathcal{F}} \longrightarrow \hat{f}(\xi)+\alpha\hat{g}(\xi)$$

Pandora's Box

$$\frac{\mathrm{d}}{\mathrm{d}x} f(x) \longrightarrow \boxed{\mathcal{F}} \longrightarrow i\xi \hat{f}(\xi)$$

Pandora's Box

$$\hat{f}(\xi) \longrightarrow \boxed{\mathcal{F}^{-1}} \longrightarrow f(x)$$

Pandora's Box

$$\left[\frac{\mathrm{d}^2}{\mathrm{d}x^2} + \gamma\frac{\mathrm{d}}{\mathrm{d}x} + \omega_0^2\right] y(x) = f(x)$$

$$\downarrow$$

$$\boxed{\mathcal{F}}$$

$$\downarrow$$

$$\left[-\xi^2 + i\gamma\xi + \omega_0^2\right] \hat{y}(\xi) = \hat{f}(\xi)$$

Box Proposal

$$\mathcal{F}[f](\xi) = \hat{f}(\xi) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{+\infty} f(x) e^{-ix\xi} \, \mathrm{d}x$$

$$\mathcal{F}^{-1}[\hat{f}](x) = f(x) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{+\infty} \hat{f}(\xi) e^{ix\xi} \, \mathrm{d}\xi$$

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Quality Control

$$(\widehat{f + \alpha g})(\xi) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{+\infty} (f(x) + \alpha g(x))\, e^{-ix\xi}\, \mathrm{d}x$$

$$\Downarrow$$

$$(\widehat{f + \alpha g})(\xi) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{+\infty} f(x) e^{-ix\xi}\, \mathrm{d}x + \frac{\alpha}{\sqrt{2\pi}} \int_{-\infty}^{+\infty} g(x) e^{-ix\xi}\, \mathrm{d}x$$

$$\Downarrow$$

$$(\widehat{f + \alpha g})(\xi) = \hat{f}(\xi) + \alpha \hat{g}(\xi)$$

Quality Control

$$\widehat{(f+\alpha g)}(\xi) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{+\infty} \left(f(x) + \alpha g(x)\right) e^{-ix\xi} \, \mathrm{d}x$$

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$$\widehat{(f+\alpha g)}(\xi) = \hat{f}(\xi) + \alpha\hat{g}(\xi)$$

Quality Control

$$\widehat{f'}(\xi) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{+\infty} f'(x) e^{-ix\xi} \, \mathrm{d}x$$

$$\Downarrow$$

$$\widehat{f'}(\xi) = \left. \frac{f(x) e^{-ix\xi}}{\sqrt{2\pi}} \right|_{-\infty}^{+\infty} + i\xi \cdot \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{+\infty} f(x) e^{-ix\xi} \, \mathrm{d}x$$

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$$\widehat{f'}(\xi) = i\xi \widehat{f}(\xi)$$

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$$\Downarrow$$

$$\widehat{f'}(\xi) = i\xi \widehat{f}(\xi)$$

Quality Control

The inverse does work

for appropriate functions

and, sometimes, the Fourier Transform of a function is not in the same set as the original function, but let's forget about this since we do not know a decent theory of integration

Playing Around With Our New Toy

Fourier Transforming

$$f(t) = \cos(\omega_0 t)e^{-\pi t^2}$$

$$\widehat{f}(\omega) = \frac{e^{-\frac{(\omega-\omega_0)^2}{4\pi}} + e^{-\frac{(\omega+\omega_0)^2}{4\pi}}}{2\sqrt{2\pi}}$$

$$\omega = 2\pi\nu$$

Fourier Transforming

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Fourier Transforming

$$f(t) = \cos(2\pi\nu_0 t)e^{-\pi t^2}$$

$$\widehat{f}(\nu) = \frac{e^{-\pi(\nu-\nu_0)^2} + e^{-\pi(\nu+\nu_0)^2}}{2\sqrt{2\pi}}$$

Fourier Transforming

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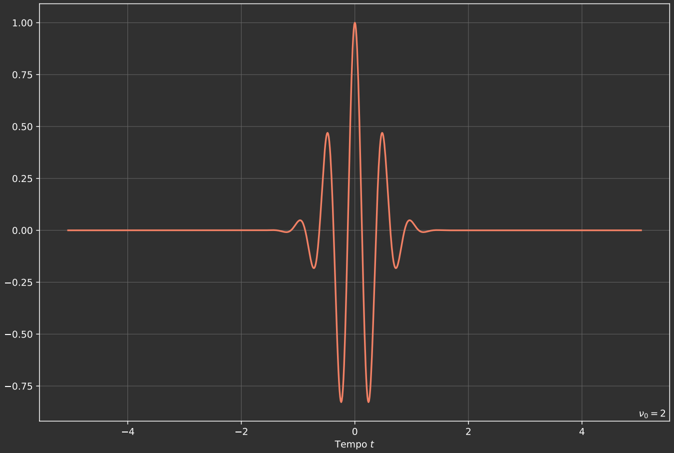

Fourier Transforming

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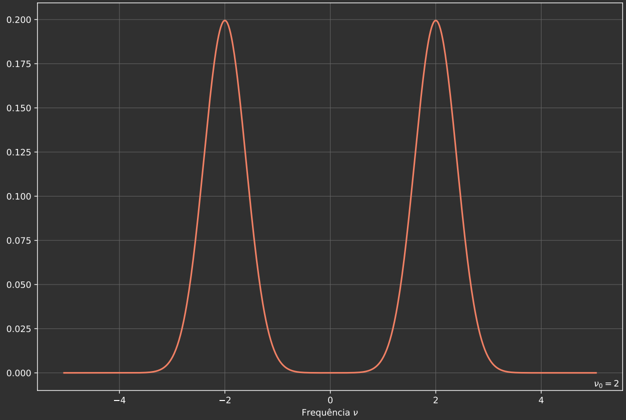

A Harder Example

$$f(t) = e^{i\omega_0 t} = \cos(\omega_0 t) + i\sin(\omega_0 t)$$

$$\widehat{f}(\omega) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{+\infty} e^{i\omega_0 t} e^{-i\omega t} \, \mathrm{d}t$$

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$$f(t) = e^{i\omega_0 t} = \cos(\omega_0 t) + i\sin(\omega_0 t)$$

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The Mathematical Moonwalk

$$f(t) = e^{i\omega_0 t}$$

$$e^{i\omega_0 t} = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{+\infty} \widehat{f}(\omega) e^{i\omega t} \, \mathrm{d}\omega$$

$$\widehat{f}(\omega) = \sqrt{2\pi}\delta\left(\omega - \omega_0\right)$$

The Mathematical Moonwalk

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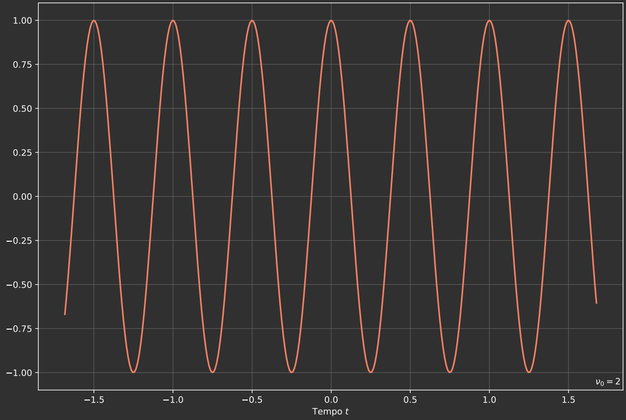
$$\widehat{f}(\omega) = \sqrt{2\pi} \delta \left(\omega - \omega_0\right)$$

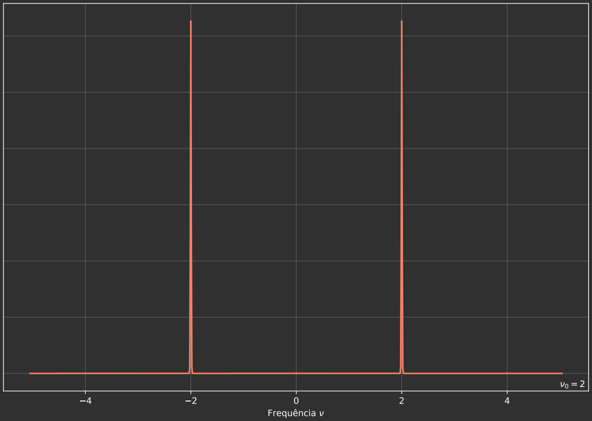
Cosines

$$f(t) = \cos(\omega_0 t) = \frac{e^{i\omega_0 t} + e^{-i\omega_0 t}}{2}$$



Cosines

$$\widehat{f}(\omega) = \sqrt{\frac{\pi}{2}}\left(\delta\left(\omega - \omega_0\right) + \delta\left(\omega + \omega_0\right)\right)$$



Fourier's Physics Playground
Maxwell's Electrodynamics

In the beggining, God said:

$$\begin{cases} \nabla \cdot \mathbf{E} = \dfrac{\rho}{\epsilon_0} \\ \nabla \cdot \mathbf{B} = 0 \\ \nabla \times \mathbf{E} = -\dfrac{\partial \mathbf{B}}{\partial t} \\ \nabla \times \mathbf{B} = \mu_0 \mathbf{J} + \mu_0 \epsilon_0 \dfrac{\partial \mathbf{E}}{\partial t} \end{cases}$$

and there was light!

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and there was light!

Too hard, let's try something different

$$\begin{cases} \mathbf{E} = -\boldsymbol{\nabla} V - \dfrac{\partial \mathbf{A}}{\partial t} \\ \mathbf{B} = \boldsymbol{\nabla} \times \mathbf{A} \end{cases}$$

Wave Equations

$$\begin{cases} \nabla^2 V - \dfrac{1}{c^2}\dfrac{\partial^2 V}{\partial t^2} = -\dfrac{\rho}{\epsilon_0} \\ \nabla^2 \mathbf{A} - \dfrac{1}{c^2}\dfrac{\partial^2 \mathbf{A}}{\partial t^2} = -\mu_0 \mathbf{J} \end{cases}$$

All Wave Equations In One

$$\nabla^2\psi(\mathbf{r},t) - \frac{1}{c^2}\frac{\partial^2\psi}{\partial t^2}(\mathbf{r},t) = -g(\mathbf{r},t)$$

Fourier's Opinion

$$\widehat{g}(\mathbf{r},\omega) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{+\infty} g(\mathbf{r},t) e^{-i\omega t}\,\mathrm{d}t$$

$$g(\mathbf{r},t) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{+\infty} \widehat{g}(\mathbf{r},\omega) e^{i\omega t}\,\mathrm{d}\omega$$

Fourier's Opinion

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Fourier's Opinion

$$\nabla^2\psi(\mathbf{r},t) - \frac{1}{c^2}\frac{\partial^2\psi}{\partial t^2}(\mathbf{r},t) = -g(\mathbf{r},t)$$

$$\nabla^2\widehat{\psi}(\mathbf{r},\omega) + \frac{\omega^2}{c^2}\widehat{\psi}(\mathbf{r},\omega) = -\widehat{g}(\mathbf{r},\omega)$$

Green Function

$$L\phi(\mathbf{r}) = -s(\mathbf{r})$$

$$LG(\mathbf{r}-\mathbf{r}') = -\delta\left(\mathbf{r}-\mathbf{r}'\right)$$

$$\phi(\mathbf{r}) = \int G(\mathbf{r}-\mathbf{r}')s(\mathbf{r}')\,\mathrm{d}\tau'$$

$$L\phi(\mathbf{r}) = \int LG(\mathbf{r}-\mathbf{r}')s(\mathbf{r}')\,\mathrm{d}\tau' = -\int \delta\left(\mathbf{r}-\mathbf{r}'\right)s(\mathbf{r}')\,\mathrm{d}\tau' = -s(\mathbf{r})$$

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One At a Time

$$\nabla^2\widehat{\psi}(\mathbf{r},\omega) + \frac{\omega^2}{c^2}\widehat{\psi}(\mathbf{r},\omega) = -\widehat{g}(\mathbf{r},\omega)$$

$$\nabla^2 G(\mathbf{r}-\mathbf{r}') + \frac{\omega^2}{c^2}G(\mathbf{r}-\mathbf{r}') = -\delta\left(\mathbf{r}-\mathbf{r}'\right)$$

One At a Time

$$\nabla^2 \widehat{\psi}(\mathbf{r}, \omega) + \frac{\omega^2}{c^2} \widehat{\psi}(\mathbf{r}, \omega) = -\widehat{g}(\mathbf{r}, \omega)$$

$$\nabla^2 G(\mathbf{r} - \mathbf{r}') + \frac{\omega^2}{c^2} G(\mathbf{r} - \mathbf{r}') = -\delta\left(\mathbf{r} - \mathbf{r}'\right)$$

Solution for $\mathbf{r} - \mathbf{r}' \neq \mathbf{0}$

$$\frac{1}{r}\frac{\mathrm{d}^2(rG)}{\mathrm{d}r^2} + k^2 G = 0$$

$$G(r) = \frac{A}{r}e^{\pm ikr}$$

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Recovering 0 Psychological Trauma

$$\nabla^2 G(\mathbf{r}-\mathbf{r}') + \frac{\omega^2}{c^2} G(\mathbf{r}-\mathbf{r}') = -\delta\left(\mathbf{r}-\mathbf{r}'\right)$$

$$A \int \nabla^2 \frac{1}{r}\, \mathrm{d}\tau' + 4\pi A \frac{\omega^2}{c^2} \int \frac{r^2}{r}\, \mathrm{d}r = -\int \delta\left(\mathbf{r}-\mathbf{r}'\right) \mathrm{d}\tau'$$

$$-4\pi A = -1$$

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$$-4\pi A = -1$$

Back To Our Problem

$$\widehat{\psi}(\mathbf{r},\omega) = \int G(\mathscr{r})\widehat{g}(\mathbf{r}',\omega)\,\mathrm{d}\tau'$$

$$G(\mathscr{r}) = \frac{1}{4\pi\mathscr{r}}e^{\pm ik\mathscr{r}}$$

$$\widehat{\psi}(\mathbf{r},\omega) = \frac{1}{4\pi}\int \frac{\widehat{g}(\mathbf{r}',\omega)e^{\pm ik\mathscr{r}}}{\mathscr{r}}\,\mathrm{d}\tau'$$

Back To Our Problem

$$\widehat{\psi}(\mathbf{r},\omega) = \int G(\mathscr{r})\widehat{g}(\mathbf{r}',\omega)\,\mathrm{d}\tau'$$

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Actually Solving Our Problem

$$\psi(\mathbf{r},t) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{+\infty} \widehat{\psi}(\mathbf{r},\omega) e^{i\omega t}\,\mathrm{d}\omega$$

$$\psi(\mathbf{r},t) = \frac{1}{4\pi\sqrt{2\pi}} \iint \frac{\widehat{g}(\mathbf{r}',\omega) e^{i\omega t \pm i\omega \frac{\mathscr{r}}{c}}}{\mathscr{r}}\,\mathrm{d}\omega\,\mathrm{d}\tau'$$

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$$\psi(\mathbf{r}, t) = \frac{1}{4\pi} \int \frac{g(\mathbf{r}', t \pm \frac{\mathscr{r}}{c})}{\mathscr{r}} \,\mathrm{d}\tau'$$

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Actually Solving Our Problem

$$\psi(\mathbf{r},t) = \frac{1}{4\pi\sqrt{2\pi}} \iint \frac{\widehat{g}(\mathbf{r}',\omega)e^{i\omega\left(t\pm\frac{\mathscr{r}}{c}\right)}}{\mathscr{r}}\,\mathrm{d}\omega\,\mathrm{d}\tau'$$

$$\psi(\mathbf{r},t) = \frac{1}{4\pi}\int \frac{g\left(\mathbf{r}',t-\frac{\mathscr{r}}{c}\right)}{\mathscr{r}}\,\mathrm{d}\tau'$$

Back at Maxwell's

$$V(\mathbf{r}, t) = \frac{1}{4\pi\epsilon_0} \int \frac{\rho(\mathbf{r}', t - \frac{\mathscr{r}}{c})}{\mathscr{r}} \, \mathrm{d}\tau'$$

$$\mathbf{A}(\mathbf{r}, t) = \frac{\mu_0}{4\pi} \int \frac{\mathbf{J}(\mathbf{r}', t - \frac{\mathscr{r}}{c})}{\mathscr{r}} \, \mathrm{d}\tau'$$

One Last Step

$$\begin{cases} \mathbf{E} = -\boldsymbol{\nabla} V - \dfrac{\partial \mathbf{A}}{\partial t} \\ \mathbf{B} = \boldsymbol{\nabla} \times \mathbf{A} \end{cases}$$

Jefimenko Equations

$$\mathbf{E}(\mathbf{r},t) = \frac{1}{4\pi\epsilon_0} \int \frac{\hat{\boldsymbol{\mathscr{r}}}}{\mathscr{r}^2} \left\lfloor \rho \right\rfloor + \frac{\hat{\boldsymbol{\mathscr{r}}}}{c\,\mathscr{r}} \left\lfloor \frac{\partial \rho}{\partial t} \right\rfloor - \frac{1}{c^2\,\mathscr{r}} \left\lfloor \frac{\partial \mathbf{J}}{\partial t} \right\rfloor \mathrm{d}\tau'$$

$$\mathbf{B}(\mathbf{r},t) = \frac{\mu_0}{4\pi} \int \left(\frac{1}{\mathscr{r}^2} \left\lfloor \mathbf{J} \right\rfloor + \frac{1}{c\,\mathscr{r}} \left\lfloor \frac{\partial \mathbf{J}}{\partial t} \right\rfloor \right) \times \hat{\boldsymbol{\mathscr{r}}}\, \mathrm{d}\tau'$$

Fourier's Physics Playground
Heisenberg's Uncertainty Principle

Position and Momentum

$$\psi(x) = \langle x|\psi\rangle = \int \langle x|k\rangle \langle k|\psi\rangle \, \mathrm{d}k = \frac{1}{\sqrt{2\pi}} \int e^{ikx}\psi(k) \, \mathrm{d}k$$

$$\psi(k) = \langle k|\psi\rangle = \int \langle k|x\rangle \langle x|\psi\rangle \, \mathrm{d}x = \frac{1}{\sqrt{2\pi}} \int e^{-ikx}\psi(x) \, \mathrm{d}x$$

Position and Momentum

$$\psi(x) = \langle x|\psi\rangle = \int \langle x|k\rangle \langle k|\psi\rangle \, \mathrm{d}k = \frac{1}{\sqrt{2\pi}} \int e^{ikx} \psi(k) \, \mathrm{d}k$$

$$\psi(k) = \langle k|\psi\rangle = \int \langle k|x\rangle \langle x|\psi\rangle \, \mathrm{d}x = \frac{1}{\sqrt{2\pi}} \int e^{-ikx} \psi(x) \, \mathrm{d}x$$

Position and Momentum (but weirder)

$$\begin{cases} X\left|\psi\right\rangle = x\psi(x) \\ K\left|\psi\right\rangle = -i\dfrac{\partial\psi}{\partial x}(x) \end{cases}$$

$$\begin{cases} X\left|\psi\right\rangle = -i\dfrac{\partial\psi}{\partial k}(k) \\ K\left|\psi\right\rangle = k\psi(k) \end{cases}$$

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$$\begin{cases} X\left|\psi\right\rangle = -i\dfrac{\partial\psi}{\partial k}(k) \\ K\left|\psi\right\rangle = k\psi(k) \end{cases}$$

Fourier Diplomacy

$$|x\rangle \xleftrightarrow[\mathcal{F}^{-1}]{\mathcal{F}} |k\rangle$$

Fourier Uncertainty

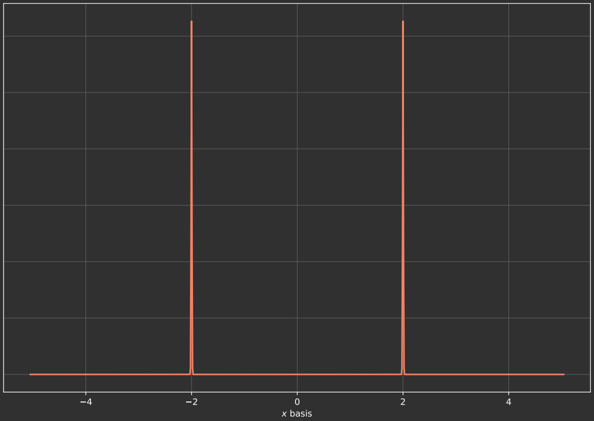
1. $\psi(x)$: what is x?
2. $\psi(k)$: what is k?

Fourier Uncertainty

1. $\psi(x)$: what is x?
2. $\psi(k)$: what is k?

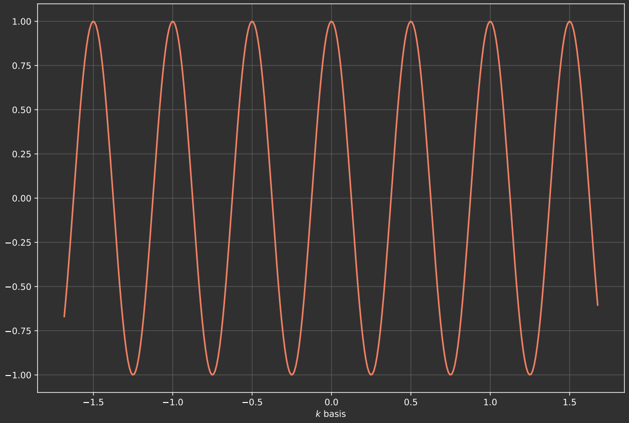
Definite Position

$$\psi(x) = \sqrt{\frac{\pi}{2}} \left(\delta\left(x - x_0\right) + \delta\left(x + x_0\right)\right)$$



Undefinite Momentum

$$\psi(k) = \cos(x_0 k)$$



Uncertainty Relation

$$\sigma_x \sigma_p \geq \frac{\hbar}{2}$$

The uncertainty relation is a consequence of the general fact that anything narrow in one space is wide in the transform space and vice versa. So if you are a 45 kg weakling and are taunted by a 270 kg bully, just ask him to step into momentum space!

Ramamurti Shankar

Acknowledgments

The author is extremely thankful to Prof. Antônio F. R. T. Piza for the short, yet wonderful, conversations about this seminar.

References

- de Figueiredo, D. G. *Análise de Fourier e Equações Diferenciais Parciais.* 5th ed. (IMPA, 2018).
- Fleming, H. *George Green e Suas Funções.* http://www.hfleming.com/green.pdf.
- Panofsky, W. K. H. & Phillips, M. *Classical Electricity and Magnetism.* 2nd ed. (Addison-Wesley Publishing Company, Inc., 1962).
- Shankar, R. *Principles of Quantum Mechanics.* 2nd ed. (Springer, 1994).

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