TRL algorithm to de-embed a RF test fixture

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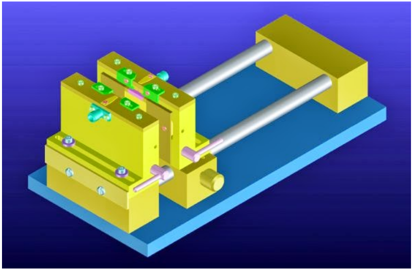
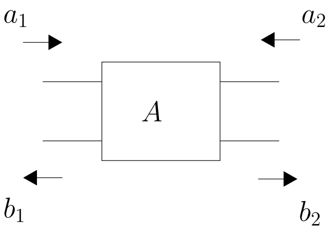
1. TRL
   - Standards
   - THRU and LINE Measurements
   - T_{OUT} parameters : $\left(\frac{T_{12}}{T_{11}}\right)$ and $\left(\frac{T_{21}}{T_{22}}\right)$
   - $\overline{T}_{IN}$ parameters : $\left(\frac{\overline{T}_{12}}{\overline{T}_{11}}\right)$ and $\left(\frac{\overline{T}_{21}}{\overline{T}_{22}}\right)$
   - The THRU equality : $\left(\frac{T_{11}}{\overline{T}_{11}}\right)$ and $\left(\frac{T_{21}}{\overline{T}_{22}}\right)$
   - The REFLECT equality : Extracting $\left(\frac{\overline{T}_{21}}{\overline{T}_{11}}\right)$
2. Reciprocity
3. Scilab Code
   - Presentation
   - Example
   - Insights

Motivations for this talk



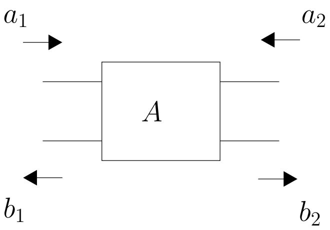
- De-embedding of a test-fixture measured in the coaxial reference planes ;
- Perform a TRL calibration when the VNA provides some ill conditioned solutions ("Reflect" checking not correct) ;
- Offer a open, complete and ready-to-use source code for educational purpose.

Some definitions : [S] parameters



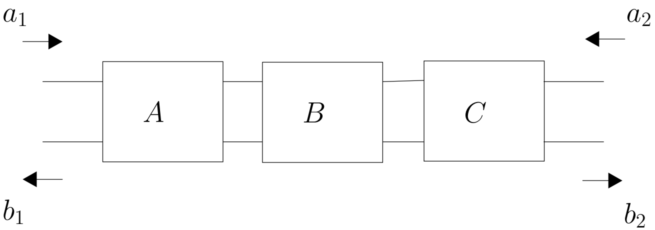
$$\begin{pmatrix} b_1 \\ b_2 \end{pmatrix} = \begin{bmatrix} S_{11} & S_{12} \\ S_{21} & S_{22} \end{bmatrix} \cdot \begin{pmatrix} a_1 \\ a_2 \end{pmatrix} \tag{1}$$

Some definitions : [T] parameters



$$\begin{pmatrix} a_1 \\ b_1 \end{pmatrix} = \begin{bmatrix} T_{11} & T_{12} \\ T_{21} & T_{22} \end{bmatrix} \cdot \begin{pmatrix} b_2 \\ a_2 \end{pmatrix} \tag{2}$$

Some definitions : [T] parameters



$$[T_{Total}] = [T_A] \cdot [T_B] \cdot [T_C] \tag{3}$$

Conversions between [S] and [T]

- $[S]$ to $[T]$

$$[T] = \begin{bmatrix} -\frac{1}{S_{21}} & -\frac{S_{22}}{S_{21}} \\ \frac{S_{11}}{S_{21}} & \frac{S_{21}\cdot S_{12}-S_{11}\cdot S_{22}}{S_{21}} \end{bmatrix} \tag{4}$$

- $[T]$ to $[S]$

$$[S] = \begin{bmatrix} -\frac{T_{21}}{T_{11}} & \frac{T_{11}\cdot T_{22}-T_{12}\cdot T_{21}}{T_{11}} \\ \frac{1}{T_{11}} & -\frac{T_{12}}{T_{11}} \end{bmatrix} \tag{5}$$

Standards for the TRL algorithm

- THRU : totally known

$$\left[T_{THRU}^{Std}\right] = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \tag{6}$$

- REFLECT : unknown

$$\mathrm{Sgn}\left(\Re\left\{\Gamma_{REFLECT}^{Std}\right\}\right) = \pm 1 \tag{7}$$

- LINE : partially known

$$\left[T_{LINE}^{Std}\right] = \begin{bmatrix} e^{-\gamma \cdot l} & 0 \\ 0 & e^{+\gamma \cdot l} \end{bmatrix} \tag{8}$$

Measuring the THRU

$$\left[T_{THRU}^{Meas}\right] = [T_{IN}] \cdot \left[T_{THRU}^{Std}\right] \cdot [T_{OUT}] \tag{9}$$

$$[T_{IN}]^{-1} \cdot \left[T_{THRU}^{Meas}\right] = \left[T_{THRU}^{Std}\right] \cdot [T_{OUT}] \tag{10}$$

$$[T_{IN}]^{-1} \cdot \left[T_{THRU}^{Meas}\right] = [T_{OUT}] \tag{11}$$

$$[T_{IN}]^{-1} = [\overline{T_{IN}}]$$

$$[\overline{T_{IN}}] \cdot \left[T_{THRU}^{Meas}\right] = [T_{OUT}] \tag{12}$$

Measuring the LINE

$$\left[T_{LINE}^{Meas}\right] = [T_{IN}] \cdot \left[T_{LINE}^{Std}\right] \cdot [T_{OUT}] \tag{13}$$

$$[T_{IN}]^{-1} \cdot \left[T_{LINE}^{Meas}\right] = \left[T_{LINE}^{Std}\right] \cdot [T_{OUT}] \tag{14}$$

$$\left[\overline{T_{IN}}\right] \cdot \left[T_{LINE}^{Meas}\right] = \begin{bmatrix} e^{-\gamma \cdot l} & 0 \\ 0 & e^{+\gamma \cdot l} \end{bmatrix} \cdot [T_{OUT}] \tag{15}$$

OUTPUT : Defining the [M] matrix

Equation (15) is :

$$\begin{bmatrix} \overline{T}_{11} & \overline{T}_{12} \\ \overline{T}_{21} & \overline{T}_{22} \end{bmatrix} \cdot \left[T_{LINE}^{Meas} \right] = \begin{bmatrix} T_{11} \cdot e^{-\gamma \cdot l} & T_{12} \cdot e^{-\gamma \cdot l} \\ T_{21} \cdot e^{+\gamma \cdot l} & T_{22} \cdot e^{+\gamma \cdot l} \end{bmatrix} \tag{16}$$

(12) in (16) give :

$$\begin{bmatrix} T_{11} & T_{12} \\ T_{21} & T_{22} \end{bmatrix} \cdot \left[T_{THRU}^{Meas} \right]^{-1} \cdot \left[T_{LINE}^{Meas} \right] = \begin{bmatrix} T_{11} \cdot e^{-\gamma \cdot l} & T_{12} \cdot e^{-\gamma \cdot l} \\ T_{21} \cdot e^{+\gamma \cdot l} & T_{22} \cdot e^{+\gamma \cdot l} \end{bmatrix} \tag{17}$$

or

$$\begin{bmatrix} T_{11} & T_{12} \\ T_{21} & T_{22} \end{bmatrix} \cdot [M] = \begin{bmatrix} T_{11} \cdot e^{-\gamma \cdot l} & T_{12} \cdot e^{-\gamma \cdot l} \\ T_{21} \cdot e^{+\gamma \cdot l} & T_{22} \cdot e^{+\gamma \cdot l} \end{bmatrix} \tag{18}$$

with

$$[M] = \left[T_{THRU}^{Meas} \right]^{-1} \cdot \left[T_{LINE}^{Meas} \right]$$

OUTPUT : TRL Equations

Equations given by (18) are :

$$T_{11} \cdot M_{11} + T_{12} \cdot M_{21} = T_{11} \cdot e^{-\gamma \cdot l} \tag{19}$$

$$T_{11} \cdot M_{12} + T_{12} \cdot M_{22} = T_{12} \cdot e^{-\gamma \cdot l} \tag{20}$$

$$T_{21} \cdot M_{11} + T_{22} \cdot M_{21} = T_{21} \cdot e^{+\gamma \cdot l} \tag{21}$$

$$T_{21} \cdot M_{12} + T_{22} \cdot M_{22} = T_{22} \cdot e^{+\gamma \cdot l} \tag{22}$$

OUTPUT : Solving $\left(\frac{T_{12}}{T_{11}}\right)$

(20) gives :

$$e^{-\gamma \cdot l} = \left(\frac{T_{11}}{T_{12}}\right) \cdot M_{12} + M_{22} \tag{23}$$

(23) in (19) gives :

$$T_{11} \cdot M_{11} + T_{12} \cdot M_{21} = T_{11} \cdot \left[\left(\frac{T_{11}}{T_{12}}\right) \cdot M_{12} + M_{22}\right] \tag{24}$$

$$M_{11} + \left(\frac{T_{12}}{T_{11}}\right) \cdot M_{21} = \left(\frac{T_{11}}{T_{12}}\right) \cdot M_{12} + M_{22} \tag{25}$$

$$\left(\frac{T_{12}}{T_{11}}\right)^2 \cdot M_{21} + \left(\frac{T_{12}}{T_{11}}\right) \cdot (M_{11} - M_{22}) - M_{12} = 0 \tag{26}$$

OUTPUT : Solving $\left(\frac{T_{22}}{T_{21}}\right)$

(21) gives :

$$e^{+\gamma \cdot l} = M_{11} + \left(\frac{T_{22}}{T_{21}}\right) \cdot M_{21} \tag{27}$$

(27) in (22) gives :

$$T_{21} \cdot M_{12} + T_{22} \cdot M_{22} = T_{22} \cdot \left[\left(\frac{T_{22}}{T_{21}}\right) \cdot M_{21} + M_{11}\right] \tag{28}$$

$$M_{22} + \left(\frac{T_{21}}{T_{22}}\right) \cdot M_{12} = \left(\frac{T_{22}}{T_{21}}\right) \cdot M_{21} + M_{11} \tag{29}$$

$$\left(\frac{T_{22}}{T_{21}}\right)^2 \cdot M_{21} + \left(\frac{T_{22}}{T_{21}}\right) \cdot (M_{11} - M_{22}) - M_{12} = 0 \tag{30}$$

OUTPUT : $\left(\frac{T_{12}}{T_{11}}\right)$ and $\left(\frac{T_{22}}{T_{21}}\right)$

$$X^2 \cdot M_{21} + X \cdot [M_{11} - M22] - M_{12} \tag{31}$$

This polynom has 2 solutions : $\left(\frac{T_{12}}{T_{11}}\right)$ and $\left(\frac{T_{22}}{T_{21}}\right)$

Usually, $\left|\frac{T_{12}}{T_{11}}\right| < \left|\frac{T_{22}}{T_{21}}\right|$

If we consider the following polynom :

$$X^2 \cdot M_{12} + X \cdot [M_{22} - M11] - M_{21} \tag{32}$$

Then the 2 solutions are $\left(\frac{T_{11}}{T_{12}}\right)$ and $\left(\frac{T_{21}}{T_{22}}\right)$

INPUT : Defining the [N] matrix

Equation (15) is :

$$\begin{bmatrix} \overline{T}_{11} & \overline{T}_{12} \\ \overline{T}_{21} & \overline{T}_{22} \end{bmatrix} \cdot \left[T_{LINE}^{Meas} \right] = \begin{bmatrix} e^{-\gamma \cdot l} & 0 \\ 0 & e^{+\gamma \cdot l} \end{bmatrix} \cdot \begin{bmatrix} T_{11} & T_{12} \\ T_{21} & T_{22} \end{bmatrix} \quad (33)$$

(12) in (33) gives :

$$\begin{bmatrix} \overline{T}_{11} & \overline{T}_{12} \\ \overline{T}_{21} & \overline{T}_{22} \end{bmatrix} \cdot \left[T_{LINE}^{Meas} \right] = \begin{bmatrix} e^{-\gamma \cdot l} & 0 \\ 0 & e^{+\gamma \cdot l} \end{bmatrix} \cdot \begin{bmatrix} \overline{T}_{11} & \overline{T}_{12} \\ \overline{T}_{21} & \overline{T}_{22} \end{bmatrix} \cdot \left[T_{THRU}^{Meas} \right] \quad (34)$$

or

$$\begin{bmatrix} \overline{T}_{11} & \overline{T}_{12} \\ \overline{T}_{21} & \overline{T}_{22} \end{bmatrix} \cdot [N] = \begin{bmatrix} \overline{T}_{11} \cdot e^{-\gamma \cdot l} & \overline{T}_{12} \cdot e^{-\gamma \cdot l} \\ \overline{T}_{21} \cdot e^{+\gamma \cdot l} & \overline{T}_{22} \cdot e^{+\gamma \cdot l} \end{bmatrix} \quad (35)$$

with

$$[N] = \left[T_{LINE}^{Meas} \right] \cdot \left[T_{THRU}^{Meas} \right]^{-1}$$

INPUT : $\left(\frac{\overline{T}_{12}}{\overline{T}_{11}}\right)$ and $\left(\frac{\overline{T}_{22}}{\overline{T}_{21}}\right)$

Equation (35) is similar to (18). Thus we can consider :

$$X^2 \cdot N_{21} + X \cdot [N_{11} - N_{22}] - N_{12} \tag{36}$$

This polynom has 2 solutions : $\left(\frac{\overline{T}_{12}}{\overline{T}_{11}}\right)$ and $\left(\frac{\overline{T}_{22}}{\overline{T}_{21}}\right)$

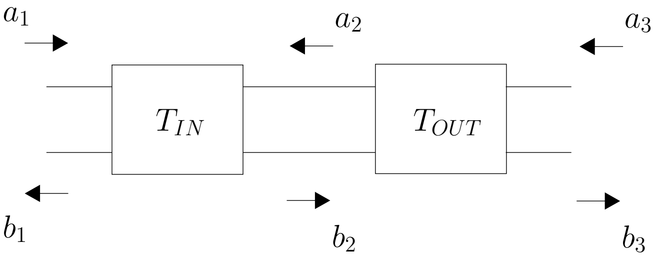
Usually, $\left|\frac{\overline{T}_{12}}{\overline{T}_{11}}\right| < \left|\frac{\overline{T}_{22}}{\overline{T}_{21}}\right|$

If we consider the following polynom :

$$X^2 \cdot N_{12} + X \cdot [N_{22} - N_{11}] - N_{21} \tag{37}$$

Then the 2 solutions are $\left(\frac{\overline{T}_{11}}{\overline{T}_{12}}\right)$ and $\left(\frac{\overline{T}_{21}}{\overline{T}_{22}}\right)$

The THRU equality



$$\begin{pmatrix} b_2 \\ a_2 \end{pmatrix} = \begin{bmatrix} \overline{T}_{11} & \overline{T}_{12} \\ \overline{T}_{21} & \overline{T}_{22} \end{bmatrix} \cdot \begin{pmatrix} a_1 \\ b_1 \end{pmatrix}$$

$$\begin{pmatrix} b_2 \\ a_2 \end{pmatrix} = \begin{bmatrix} T_{11} & T_{12} \\ T_{21} & T_{22} \end{bmatrix} \cdot \begin{pmatrix} b_3 \\ a_3 \end{pmatrix}$$

Forward mode : b_2 equality to extract $\left(\frac{T_{11}}{\overline{T}_{11}}\right)$

The b_2 equality leads us to :

$$\overline{T}_{11} \cdot a_1 + \overline{T}_{12} \cdot b_1 = T_{11} \cdot b_3 + T_{12} \cdot a_3 \tag{38}$$

And by definition, about the THRU measurement, we know :

$$S_{21}^{Meas} = \left.\frac{b_3}{a_1}\right|_{a_3=0} \text{ and } S_{11}^{Meas} = \left.\frac{b_1}{a_1}\right|_{a_3=0}$$

Thus,

$$a_1 \cdot \left(\overline{T}_{11} + \overline{T}_{12} \cdot \frac{b_1}{a_1}\right) = T_{11} \cdot b_3 \tag{39}$$

$$\overline{T}_{11} \cdot \left(1 + \left(\frac{\overline{T}_{12}}{\overline{T}_{11}}\right) \cdot S_{11}^{Meas}\right) = T_{11} \cdot S_{21}^{Meas} \tag{40}$$

$$\left(\frac{T_{11}}{\overline{T}_{11}}\right) = \frac{\left(1 + \left(\frac{\overline{T}_{12}}{\overline{T}_{11}}\right) \cdot S_{11}^{Meas}\right)}{S_{21}^{Meas}} \tag{41}$$

Reverse mode : a_2 equality to extract $\left(\frac{T_{21}}{\overline{T}_{21}}\right)$

The a_1 equality leads us to :

$$\overline{T}_{21} \cdot a_1 + \overline{T}_{22} \cdot b_1 = T_{21} \cdot b_3 + T_{22} \cdot a_3 \tag{42}$$

For the THRU measurement, we know :

$$S_{12}^{Meas} = \left.\frac{b_1}{a_3}\right|_{a_1=0} \text{ and } S_{22}^{Meas} = \left.\frac{b_3}{a_3}\right|_{a_1=0}$$

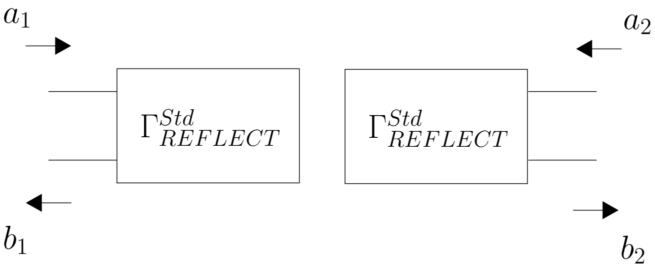
Thus,

$$b_1 \cdot \overline{T}_{22} = T_{21} \cdot b_3 + T_{21} \cdot a_3 \tag{43}$$

leads us to :

$$\left(\frac{T_{21}}{\overline{T}_{22}}\right) = \frac{S_{12}^{Meas}}{S_{22}^{Meas} + \left(\frac{T_{22}}{T_{21}}\right)} \tag{44}$$

REFLECT Measurement



$$\Gamma^{Std}_{REFLECT} = \frac{b_1}{a_1} = \frac{\overline{T}_{21} + \overline{T}_{22} \cdot S^{Meas}_{11}}{\overline{T}_{11} + \overline{T}_{12} \cdot S^{Meas}_{11}} \tag{45}$$

$$\Gamma^{Std}_{REFLECT} = \frac{b_2}{a_2} = \frac{T_{12} + T_{11} \cdot S^{Meas}_{22}}{T_{22} + T_{21} \cdot S^{Meas}_{22}} \tag{46}$$

REFLECT Equality

$$\frac{\overline{T}_{21} + \overline{T}_{22} \cdot S_{11}^{Meas}}{\overline{T}_{11} + \overline{T}_{12} \cdot S_{11}^{Meas}} = \frac{T_{12} + T_{11} \cdot S_{22}^{Meas}}{T_{22} + T_{21} \cdot S_{22}^{Meas}} \tag{47}$$

$$\frac{\overline{T}_{21}}{\overline{T}_{11}} \cdot \left(\frac{1 + \left(\frac{\overline{T}_{22}}{\overline{T}_{21}}\right) \cdot S_{11}^{Meas}}{1 + \left(\frac{\overline{T}_{12}}{\overline{T}_{11}}\right) \cdot S_{11}^{Meas}} \right) = \frac{T_{11}}{T_{21}} \cdot \left(\frac{S_{22}^{Meas} + \left(\frac{T_{12}}{T_{11}}\right)}{S_{22}^{Meas} + \left(\frac{T_{22}}{T_{21}}\right)} \right) \tag{48}$$

$$\left(\overline{T}_{21}\right)^2 \cdot \left(\frac{T_{21}}{\overline{T}_{22}}\right) \cdot \left(\frac{\overline{T}_{22}}{\overline{T}_{21}}\right) = \left(\overline{T}_{11}\right)^2 \cdot \left(\frac{T_{11}}{\overline{T}_{11}}\right) \cdot \frac{\left(\frac{S_{22}^{Meas} + \left(\frac{T_{12}}{T_{11}}\right)}{S_{22}^{Meas} + \left(\frac{T_{22}}{T_{21}}\right)}\right)}{\left(\frac{1 + \left(\frac{\overline{T}_{22}}{\overline{T}_{21}}\right) \cdot S_{11}^{Meas}}{1 + \left(\frac{\overline{T}_{12}}{\overline{T}_{11}}\right) \cdot S_{11}^{Meas}}\right)} \tag{49}$$

REFLECT Equality

$$\left(\frac{\overline{T}_{21}}{\overline{T}_{11}}\right) = \pm\sqrt{\frac{\left(\frac{T_{11}}{\overline{T}_{11}}\right)\cdot\left(\frac{S_{22}^{Meas}+\left(\frac{T_{12}}{T_{11}}\right)}{S_{22}^{Meas}+\left(\frac{T_{22}}{T_{21}}\right)}\right)}{\left(\frac{T_{21}}{\overline{T}_{22}}\right)\cdot\left(\frac{\overline{T}_{22}}{\overline{T}_{21}}\right)\cdot\left(\frac{1+\left(\frac{\overline{T}_{22}}{\overline{T}_{21}}\right)\cdot S_{11}^{Meas}}{1+\left(\frac{\overline{T}_{12}}{\overline{T}_{11}}\right)\cdot S_{11}^{Meas}}\right)}} \quad (50)$$

There are 2 solutions. We select the good one thanks to the knowledge of $\mathrm{Sgn}\left(\Re\{\Gamma_{REFLECT}^{Std}\}\right) = \pm 1$ in (45) :

$$\Gamma_{REFLECT}^{Std} = \left(\frac{\overline{T}_{21}}{\overline{T}_{11}}\right)\cdot\left(\frac{1+\left(\frac{\overline{T}_{22}}{\overline{T}_{21}}\right)\cdot S_{11}^{Meas}}{1+\left(\frac{\overline{T}_{12}}{\overline{T}_{11}}\right)\cdot S_{11}^{Meas}}\right) \quad (51)$$

TRL Completed

The TRL algorithm is completed. We got 7 parameters from :

- The $[M]$ matrix : $\left(\frac{T_{12}}{T_{11}}\right)$ and $\left(\frac{T_{22}}{T_{21}}\right)$ from (31) ;
- The $[N]$ matrix : $\left(\frac{\overline{T}_{12}}{\overline{T}_{11}}\right)$ and $\left(\frac{\overline{T}_{22}}{\overline{T}_{21}}\right)$ from (36) ;
- The THRU equality : $\left(\frac{T_{11}}{\overline{T}_{11}}\right)$ from (41) and $\left(\frac{T_{21}}{\overline{T}_{22}}\right)$ from (44) ;
- The REFLECT equality : $\left(\frac{\overline{T}_{21}}{\overline{T}_{11}}\right)$ from (50) ;

It is suffisent for $[S]$ parameters de-embedding but not for power measurement.
We need to normalize correctly the system of equation (finding the absolute value of $\overline{T}_{11}$).
For that purpose we will consider a reciprocity assumption : $S_{21}^{IN} = S_{12}^{IN}$.

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Reciprocity assumption

We should have :

$$\left[\overline{T}_{IN}\right] = \begin{bmatrix} \frac{S_{21} \cdot S_{12} - S_{11} \cdot S_{22}}{S_{12}} & \frac{S_{22}}{S_{12}} \\ -\frac{S_{11}}{S_{12}} & \frac{1}{S_{12}} \end{bmatrix} \tag{52}$$

Thus the reciprocity assumption ($S_{21} = S_{12}$) leds to :

$$\overline{T}_{11} \cdot \overline{T}_{22} - \overline{T}_{12} \cdot \overline{T}_{21} = 1 \tag{53}$$

Reciprocity assumption

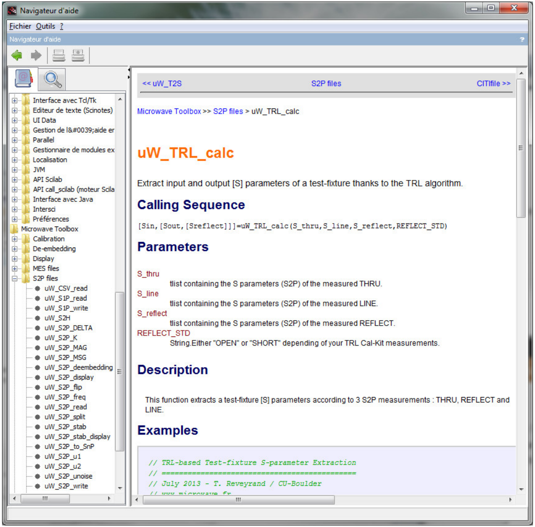
We can obtain from TRL the complete $[\overline{T}_{IN}]$ and $[T_{OUT}]$ matrix from an arbitrary value of $\overline{T}_{11}$. Those matrix has to be multiplied by K in order to fullfill equation (53) such as :

$$K^2 = \frac{1}{\overline{\overline{T}}_{11} \cdot \overline{\overline{T}}_{22} - \overline{\overline{T}}_{12} \cdot \overline{\overline{T}}_{21}} \tag{54}$$

$$K = \pm \sqrt{\frac{1}{\overline{\overline{T}}_{11} \cdot \overline{\overline{T}}_{22} - \overline{\overline{T}}_{12} \cdot \overline{\overline{T}}_{21}}} \tag{55}$$

There are 2 solutions. The good one is selected such as the extrapolated phase of S_{21} on DC is as close as possible of zero.

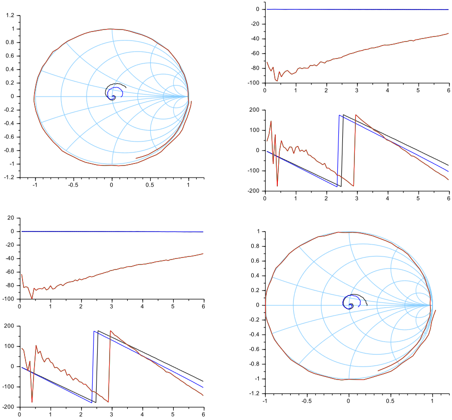
This code is now available in Scilab



http://www.microwave.fr/uW.html

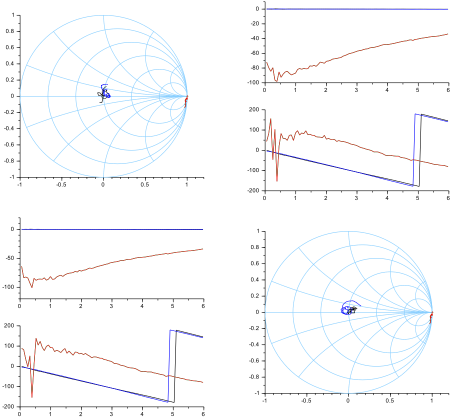
Example with an OPEN reflect

S2P Measurements : Thru (black), Line (blue) and Open (red).



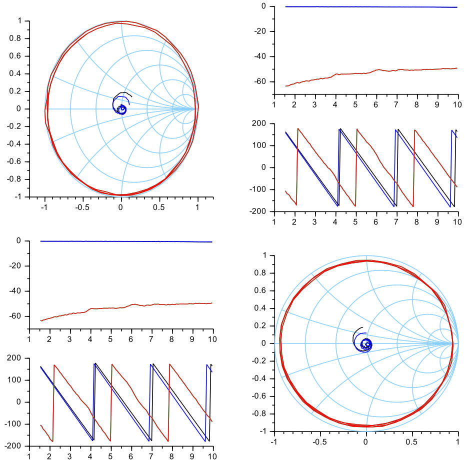
Example with an OPEN reflect

S2P Extracted : Port 1 (black), Port 2 (blue) and De-embedded open (red).



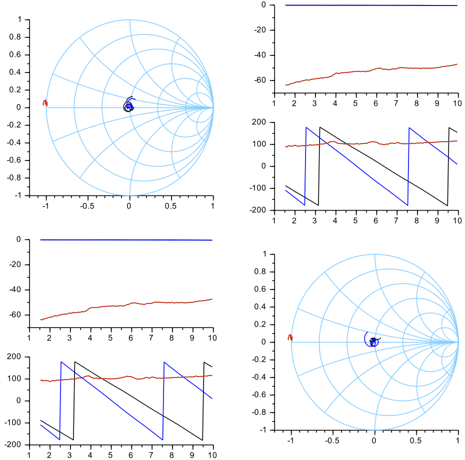
Example with an SHORT reflect

S2P Measurements : Thru (black), Line (blue) and Short (red).



Example with an SHORT reflect

S2P Extracted : Port 1 (black), Port 2 (blue) and De-embedded short (red).



Source Code : uW_TRL_calc.sci

```
function varargout=uW_TRL_calc(S_thru,S_line,S_reflect,REFLECT_STD)

T_THRU=uW_S2T(S_thru);
T_LINE=uW_S2T(S_line);

M11=[];M12=[];M21=[];M22=[];
N11=[];N12=[];N21=[];N22=[];

T_IN=list();
T_OUT=list();

K=[];

for i=1:size(T_THRU.frequency,1),
    T1=[T_THRU.T11(i),T_THRU.T12(i);T_THRU.T21(i),T_THRU.T22(i)];
    T2=[T_LINE.T11(i),T_LINE.T12(i);T_LINE.T21(i),T_LINE.T22(i)];
    M=inv(T1)*T2;
    N=T2*inv(T1);

    // Equation TRL M
    delta=(M(1,1)-M(2,2))^2-4*M(2,1)*(-M(1,2));
    X1=((M(2,2)-M(1,1))-sqrt(delta))/(2*M(2,1));
    X2=((M(2,2)-M(1,1))+sqrt(delta))/(2*M(2,1));
    Sol=[X1,X2];
    C1=Sol(find(abs(Sol)==max(abs(Sol))));  // OUTPUT - T22/T21
    C2=Sol(find(abs(Sol)==min(abs(Sol))));  // OUTPUT - T12/T11

    // Equation TRL N
    delta=(N(1,1)-N(2,2))^2-4*N(2,1)*(-N(1,2));
    X1=((N(2,2)-N(1,1))-sqrt(delta))/(2*N(2,1));
    X2=((N(2,2)-N(1,1))+sqrt(delta))/(2*N(2,1));
    Sol=[X1,X2];
    C3=Sol(find(abs(Sol)==max(abs(Sol))));  // INPUT - D/C
    C4=Sol(find(abs(Sol)==min(abs(Sol))));  // INPUT - B/A

    // Equation THRU FORWARD
    C5=(1+C4*S_thru.S11(i))/(S_thru.S21(i));    // T11/A

    // Equation THRU REVERSE
    C6=(S_thru.S12(i))/(S_thru.S22(i)+C1);      // T21/D

    // Equation Reflect
    _X=(S_reflect.S22(i)+C2)/(S_reflect.S22(i)+C1);
    _Y=(1+C3*S_reflect.S11(i))/(1+C4*S_reflect.S11(i));
    sol=sqrt((C5*_X)/(_Y*C6*C3));       // C

    A=1;B=C4;C=sol;D=C3*C;
    GAMMA_STD=(C+D*S_reflect.S11(i))/(A+B*S_reflect.S11(i));
    if ((real(GAMMA_STD)>0)&REFLECT_STD=="SHORT")|((real(GAMMA_STD)<0)&REFLECT_STD=="OPEN") then,
        sol=sol*(-1);
    end;
```

Source Code : uW_TRL_calc.sci

```
    A=1;B=C4;C=sol;D=C3*C;

    K=[K;(sqrt(1/(A*D-B*C)))];

    T_IN($+1)=inv([A,B;C,D]);
    T_OUT($+1)=[C5,C2*C5;C6*D,C1*C6*D];

end;

// === Reciprocity : K unwaping
        x=phasemag(K(2:$))-phasemag(K(1:($-1)));
        a=(x<-90)*180+(x>90)*(-180);
        a=cumsum(a,'r');
        a=[0;a];
        uwp=real(phasemag(K(1:$))+a);
        cc=pinv([S_thru.frequency/10^9,uwp*0+1])*uwp;
        uwp=uwp-cc(2)+modulo(cc(2),180);
        K=abs(K).*exp(%i*%pi*uwp/180);

// === BUILD T MATRIX
T11=[];T12=[];T21=[];T22=[];
for i=1:size(T_THRU.frequency,1),
    T11=[T11;T_IN(i)(1,1)];T12=[T12;T_IN(i)(1,2)];T21=[T21;T_IN(i)(2,1)];T22=[T22;T_IN(i)(2,2)];
end;
Port_IN=tlist(['T parameters';'frequency';'T11';'T12';'T21';'T22'],T_THRU.frequency,(1 ./K).*T11,(1 ./K).*T12,(1 ./K).*T21,(1 ./K).*T22);

T11=[];T12=[];T21=[];T22=[];
for i=1:size(T_THRU.frequency,1),
    T11=[T11;T_OUT(i)(1,1)];T12=[T12;T_OUT(i)(1,2)];T21=[T21;T_OUT(i)(2,1)];T22=[T22;T_OUT(i)(2,2)];
end;
Port_OUT=tlist(['T parameters';'frequency';'T11';'T12';'T21';'T22'],T_THRU.frequency,K.*T11,K.*T12,K.*T21,K.*T22);

// === Convert to S-param

S_IN=uW_T2S(Port_IN);
S_OUT=uW_T2S(Port_OUT);
S_R=uW_S2P_deembedding(S_reflect,S_IN,S_OUT);

      varargout=list(S_IN,S_OUT,S_R);

endfunction
```