

Distance between two hyperplanes

AK

March 14, 2015

What is the distance between two hyperplanes?

A hyperplane is a subspace with dimension $n-1$ (see <http://mathworld.wolfram.com/Hyperplane.html>). Suppose two parallel hyperplanes are defined as $\{x \in \mathbb{R}^n | (a, x) = b_1\}$ and $\{x \in \mathbb{R}^n | (a, x) = b_2\}$ where $a \in \mathbb{R}^n$ is the same vector (otherwise they are not parallel and the distance is 0).

Let us consider a line that passes through the origin and is orthogonal to both hyperplanes. This line has a as a direction vector and can be described by a parametrized equation $x = ta, t \in \mathbb{R}$.

It is to find the points x_1 and x_2 of intersection with the hyperplanes:

$$(a, at_1) = b_1 \implies t_1 = \frac{b_1}{(a, a)} \implies x_1 = \frac{b_1 a}{(a, a)}$$

$$(a, at_2) = b_2 \implies t_2 = \frac{b_2}{(a, a)} \implies x_2 = \frac{b_2 a}{(a, a)}$$

Finally, the distance between points x_1 and x_2 can be found:

$$d = \|x_1 - x_2\| = \frac{|b_1 - b_2| \|a\|}{(a, a)} = \frac{|b_1 - b_2|}{\|a\|}$$

