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Congruence 1. Find an integer x such that $4^{128} \equiv x \ mod 9$ and $0 \leq x \leq 100$.

If we begin with $4 \equiv 4 \ mod 9$ and also, $4^2 \equiv 7 \ mod 9$, then by Proposition 21, we can say:

$$4^3 \equiv 28 \ mod 9 \text{ (where 1 also works for 28)}$$

If we repeat the original process with 4^3 until we get to 4^{126}, we can then utilize the 4 and 4^2 values respectively:

$$4^{127} \equiv 4 \ mod 9$$

and finally

$$4^{128} \equiv 7 \ mod 9.$$

Congruence 2. Find an integer y such that $3^{128} \equiv y \ mod 4$ and $0 \leq y \leq 3$.

If we begin with

$$3 \equiv 3 \ mod 4,$$
$$3^2 \equiv 1 \ mod 4,$$
$$3^3 \equiv 3 \ mod 4,$$
$$3^4 \equiv 1 \ mod 4,$$

then by Proposition 21, we can say:

$$3^6 \equiv 1 \ mod 4,$$

Considering 128 is an even integer and $3^2 \equiv 1 \ mod 4$, we can do this until we get to $3^{128} \equiv 1 \ mod 4$.

Congruence 3. For each of the following congruence's, find integers x_i such that $0 \leq x_i \leq 6$ that satisfy the congruence.