Assignment One

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Abstract

Modern Algebra HW

1 Introduction

In this assignment we will go over 6 proofs that we have recently derived from the Axioms of integer arithmetic off the assumption those Axioms are all we know. These Axioms can be found in the Notebook written by Dr. Oscar Chavez titled*Modern Algebra*

2 Problems

2.1 Prove that *a*0 =0*

- a*0=0 ==
- a*0+0=0 ==
- a*0+(a-a)=0 ==
- (a*0+a)-a=0 ==
- a(0+1)-a =0 ==
- a-a=0 ==
- 0=0 QED

We have proven a*0=0 .

2.2 Prove that -(ab)=a(-b)

We have proven a*0=0 .

- a*0=0 ==
- a(b+(-b))=0 ==
- (ab)+a(-b)=0 ==

- ab+a(-b)-ab=0-ab ==
- a(-b)=-(ab) QED

We have proven a(-b)=-(ab) .

2.3 Prove that (-a)(-b)=ab

We know a+(-a)=0 and b+(-b)=0 .

- (a+(-a))(b+(-b))=0 ==
- ab+a(-b)+(-a)b+(-a)(-b)=0 ==

We have proven a(-b)=-(ab) .

- ab+(-(ab))+(-(ab))+(-a)(-b)=0 ==
- (-(ab))+(-a)(-b)=0 ==
- (-(ab))+(ab)+(-a)(-b)=0+(ab ==
- (-a)(-b)=ab QED

We have proven (-a)(-b)=ab .

2.4 Prove For all integers a and b, if ab=0 and a does not = 0, then b=0

Case 1: if b is greater than 0

- If a does not equal zero and bis greater than 0 then ab does NOT equal 0

Case 2: if b is less than 0

- If a does not equal zero and b is less than 0 than ab does NOT equal 0

Case 3: if b = 0

- If a does not equal zero and b=0 than ab=0

Proposition proved

2.5 Prove that, for all integers a,b, and c, if ac=bc and c does not = 0, then a=b

- ac = bc ==
- ac-(bc)=bc-(bc) ==
- ac-bc=0 ==
- c(a-b)=0 ==

if c can not equal zero we have proven (a-b) must equal 0 .

- a-b=0 ==
- a=b QED

We have proven a=b.

2.6 Let $a.b$, and c be integers such that a+b=a+c. Prove b=c

- a+b=a+c ==

a has such additive inverse such that a+(-a) =0. We will add it to both sides of the equation

- a+b+(-a)=a+c+(-a) ==
- b=c QED

We have proven b=c.