

Consider a composite function $f \circ g$. Does the independent variable have to be the same in each function? That is $f(x)$ and $g(x)$?

Answer: you can choose any name for the independent variable. What counts here is that the composition is defined whenever the image of g is contained in the domain of f . For instance imagine that g maps the number of square meters of a flat in Venice into the price of such a flat per month. Then f should act on the prices of flats per month (for instance f may "send" a price of a flat per month into the salary that you are expected to earn).

In your example $f(x) = x^2 + 4$ and $g(t) = 4t - 5$ you could have written also $f(x) = x^2 + 4$ and $g(x) = 4x - 5$. You may also write (names of the variables are not important!)

$$\forall z \quad f \circ g(z) = f(g(z)) = f(4z - 5) = (4z - 5)^2 + 4.$$